Answer Sheet to the Written Exam

Corporate Finance and Incentives

February 2021

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "After completing the course, the student is expected to be able to:

Knowledge:

1. Identify, describe and discuss financial problems encountered by firms and investors,
2. Account for and understand the core models and methodologies in the field of Financial Economics,
3. Define the core concepts of Financial Economics,
4. Criticize and reflect upon the main models in Finance, relating them to current issues in financial markets and corporate finance.

Skills:

1. Select and apply core models and methodologies to analyse standard problems in Finance, partly using Excel,
2. Master the analysis of given problems, assessing models and results, putting results into perspective,
3. Argue about financial problems and issues in a scientific and professional manner, drawing upon the relevant knowledge of the field.

Competencies:

1. Bring into play the achieved knowledge and skills on new formal problems,
2. Select and evaluate solutions to complex, unpredictable situations in financial markets or corporations,
3. Approach more advanced models, methodologies and topics in Finance."

Problems 1–3 are particularly focused on knowledge points 1–3, skills 1–3, competencies 1 and 2. Problem 4 emphasizes knowledge points 1–4, skill 3, and competencies 1 and 3.

Some numerical calculations may differ slightly depending on the commands chosen for computation, so a little slack is allowed when grading the answers.

Problem 1 (CAPM 30%)

We provide you with 60 monthly return observations for three assets (TSLA, GS and MCD) in a separate xls file. The returns are stated in percent. Use the data and Excel to complete the following tasks:

1) *Compute the sample variance-covariance matrix and sample mean of the returns for the three assets.*

Computation yields

$$\mu = E(r) = \begin{pmatrix} 5.800 \\ 0.934 \\ 1.349 \end{pmatrix} \tag{1}$$

where r is the vector of returns for *TSLA, GS* and *MCD*.

The sample covariance matrix looks like

$$\Sigma = \text{Cov}(r) \begin{pmatrix} 368.677 & 59.462 & 38.302 \\ 59.462 & 75.250 & 15.906 \\ 38.320 & 15.906 & 24.199 \end{pmatrix}. \tag{2}$$

2) *Find the minimum variance portfolio weights of the risky assets and calculate the expectation and volatility of this portfolio's return.*

The variance of a portfolio with weights w is

$$\sigma_{pf}^2 = w'\Sigma w.$$

The minimum variance portfolio weights are the minimizing argument of

$$w_{\text{mvp}} = \arg\min w'\Sigma w \text{ s.t. } w'1 = 1.$$

w_{mvp} can be derived analytically as

$$w_{\text{mvp}} = \frac{1}{\iota'\Sigma^{-1}\iota}\Sigma^{-1}\iota$$

where ι is a vector of ones. In our case, we get

$$\Sigma^{-1} = \begin{pmatrix} 0.003 & -0.002 & -0.004 \\ -0.002 & 0.016 & -0.008 \\ -0.004 & -0.008 & 0.053 \end{pmatrix}$$

and therefore

$$w_{\text{mvp}} = \begin{pmatrix} -0.058 \\ 0.148 \\ 0.910 \end{pmatrix}.$$

The expected return of a portfolio is $E(r_{\text{pf}}) = w'\mu$ and the volatility is the square root of the variance. In the case from above this is just:

$$E(r_{\text{mvp}}) = 1.027$$

and

$$\sigma(r_{\text{mvp}}) = \sqrt{w'_{\text{mvp}}\Sigma w_{\text{mvp}}} = 4.706.$$

3) *Suppose you can invest and borrow at the risk-free rate of* $r_f = 0.3\%$. *Compute the efficient tangent portfolio weights.*

We derived the tangency portfolio weights w^* in Slide 21 of the third lecture as

$$w^* = \frac{1}{\iota'\Sigma^{-1}(\mu - r_f)}\Sigma^{-1}(\mu - r_f) = \begin{pmatrix} 0.406 \\ -0.241 \\ 0.835 \end{pmatrix}.$$

4) *Given the 3 risky assets and the risk-free rate, what is the highest possible return of an investment with standard deviation* 2%*?*

We are searching for an efficient portfolio with standard deviation 2%. Make use of the two-fund theorem to see that any portfolio that invests in the risk-free rate and the efficient tangent portfolio w^* is efficient. Further, we know that the volatility of such a portfolio is

$$\sigma_{\text{pf}} = \lambda\sigma_{w^*}$$

where λ is the fraction of wealth invested in the efficient portfolio and σ_{w^*} is the efficient tangent portfolio standard deviation. We can solve for $\lambda = 2/\sqrt{(w^{*\prime}\Sigma' w^*)} = 0.211$. Then, the portfolio return is

$$E(r_{pf}) = (1 - \lambda)r_f + \lambda E(r_{w^*}{}'\mu) = 0.3 + 0.211(w^{*\prime}\mu - r_f) = 0.923.$$

5) *Assume now the market is imperfect and it is not possible to borrow at the risk-free rate* r_f. *Instead, to borrow funds investors have to pay* $r_b = 0.5\% > r_f$. *What is the highest possible return of an investment with standard deviation* 20%*?*

First, using the same expression as in e) to solve for the required λ to generate a portfolio with standard deviation of 20%, we see that we would need to invest $\lambda = 2.108$ in the efficient tangent portfolio and thus *borrow*. However, as borrowing is not possible at the risk-free rate r_f, we need to find another portfolio which is efficient, taking the higher borrowing costs into account. To do so we can repeat exercise c) and compute

$$\tilde{w} = \frac{1}{\iota'\Sigma^{-1}(\mu - r_b)}\Sigma^{-1}(\mu - r_b) = \begin{pmatrix} 0.582 \\ -0.391 \\ 0.809 \end{pmatrix}.$$

As a result, the efficient investment portfolio invests 1.626 in the (new) efficient tangent portfolio $\tilde{w}$ and -0.626 in the risk free asset. The portfolio thus has expected returns of

$$E(r_{\text{pf}}) = r_f + 1.626E(\tilde{w}'\mu - r_f) = 6.359.$$

Problem 2 (Corporate Finance 20%)

1) In perfect capital markets, this purely financial transaction leaves the firm's value unchanged at 100.

2) The debt provides a tax shield of present value 25%40 = 10, so the firm's value rises to 110.

3) The firm value rose by $110 - 107 = 3$ less than predicted in 2), which must be the expected value of financial distress costs. See equation (16.1) in Berk and DeMarzo.

4) If creditors pay personal income tax, the firm needs to promise cash flow worth more than 40 in order to get creditors to pay 40 for the new debt. If this additional payment exceeds 7, the firm's shareholders have actually lost value from this refinancing transaction. It seems realistic that creditors could face a tax rate greater than $7/47 \approx 15\%$.

Problem 3 (Options 25%)

1) *Compute the risk-neutral probabilities for each branch of the tree.*

Risk-neutral pricing implies a martingale property for the price of the asset. Therefore, at each node i we can solve for the risk-neutral probability π_i of high returns by solving for

$$S_i = \frac{1}{1+r_f}\left(\pi_i S_i^U + (1-\pi_i)S_i^D\right) = \frac{1}{1+r_f} S_i \left(\pi_i(r_U - r_D) + r_D\right)$$

where $r_U = 1.25$ and $r_D = 0.75$ in the exercise. It becomes immediately clear that the risk-neutral probabilities are the same at each node, such that we can denote

$$\pi = \frac{(1 + r_f - r_d)}{r_U - r_D} = 0.52.$$

2) *Consider three European Call options expiring at time 2 with different strike prices:* $K_1 = 92.75, K_2 = 93.75, K_3 = 94.75$*. Compute the market values (i.e. premiums) of the Calls,* C_1, C_2*, and* C_3 *at time 0.*

We use the risk-neutral probability from a) to derive the market value of the two options. Note first, the payout structure of the three Call options: The option with strike price 92.75 pays out 1 in state S_2^M. The other options pay out nothing in that state (and none of the options is exercised in state S_2^D). In state S_2^U, all three options are exercised and have cash-flows of $63.5, 62.5$, and 61.5. The market prices of each option are then

$$C_1 = \frac{1}{(1+r_f)^2}\left(\pi^2 63.5 + 2\pi(1-\pi)1\right) = 17.321.$$
$$C_2 = \frac{1}{(1+r_f)^2}\left(\pi^2 62.5\right) = 16.567.$$
$$C_3 = \frac{1}{(1+r_f)^2}\left(\pi^2 61.5\right) = 16.302.$$

3) *What is the arbitrage-free value of a butterfly spread (simultaneously buying Call options with strike K_1 and K_3 and going short two Call options with strike K_2)? What are the cash flows at time 2 in each state if you finance the portfolio by borrowing at the risk-free rate?*

The market price is simply the sum of $C_1 - 2C_2 + C_3 = 0.489$. If you finance the butterfly spread by borrowing 0.489 at time 0, the resulting (negative) cash flow at time 2 is $0.489(1+r_f)^2 = 0.4988$. As a result, the portfolio pays out -0.4988 in state S_2^U, $1-0.4988$ in state S_2^M and -0.4988 in state S_2^D.

4) *Argue why the (risk-neutral) expected payoff of the portfolio in question 3) (buying the butterfly spread and financing at the risk-free rate) at time 2 has to be zero.*

This holds simply because the market is arbitrage-free. If the risk-neutral expected return would not be zero then a (self-financing) portfolio would generate positive expected returns which corresponds to an arbitrage opportunity. Such an existence was ruled out by the fundamental theorem of asset pricing.

Problem 4 (Various Themes 25%)

1) Page 412 in Berk and DeMarzo defines the efficient frontier and illustrates the change when more stocks are included in the portfolio.

2) A good answer could discuss points such as the following. In theory, investors and firms should care about real interest rates. Nominal rates are not so important, with the exception that cash may be a superior alternative to investments with negative risk-adjusted return. In reality, investors and firms may be swayed by low interest rates to finance more (risky) activities: more projects may be deemed to have positive net present value. If this is due to a systematic undervaluation of the risk, the bank may have good reasons to worry that financial risks can build up, i.e., that investors may fail to get what they expect.

3) The claim appears in the text on pages 657 and 658 in Berk and DeMarzo. See the explanation and discussion in section 17.5.